Partial exclusivity

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Abstract

We investigate the anticompetitive effects of pre-auction negotiations in selling and procurement situations. Assuming (in the selling case) that the seller and an "incumbent" buyer can move before valuations are learnt, we show that they have a joint incentive to arrange for themselves the option of entering into exclusive negotiations after uncertainty is resolved. In equilibrium, an auction takes place with an endogenous probability that depends on the bargaining process. In that auction, the beliefs are asymmetric even when the potential buyers are ex ante symmetric.

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1 Introduction

In 2014, the French National Rugby League launched an invitation to tender for the television broadcasting rights of "Top 14" matches.[1] Shortly before before the submission deadline, the League interrupted the tendering process and granted exclusivity for the next five seasons to the incumbent right holder, Canal Plus. A major competitor, BeIN Sports, complained that it had thus been deprived of the opportunity to submit a bid in a competitive procedure, and suggested that the deal privately agreed between the rugby League and Canal Plus was anticompetitive. The French competition authority decided to suspend the contested agreement and requested the League to "proceed with a new assignment of broadcasting rights using a transparent and non-discriminatory procedure."

The seller of an indivisible good can organize the sale in many different ways. She can advertise an invitation to tender and sell it to the largest bidder. But she could also decide to have bilateral negotiations with some or all buyers before an eventual auction. Examples range from TV rights for a sport event (Football World Cup, Olympics,...) to selling a business.[2] Building on the works of Vickrey (1961), Vickrey (1962) and others, Myerson (1981) answers the question by characterizing the optimal mechanism (see Krishna (2002) 5.2 or Milgrom (2004) 4.4) from the point of view of the seller.[3] In the case where buyers are symmetric this optimal mechanism is a simple Vickrey's auction with a reserve price.

The economist' advice to the seller is thus simple: organize a Myerson's optimal auction. The seller should not bother wasting her time negotiating with any buyers. This idea that auctions do best is reinforced by Bulow and Klemperer (1996) who prove that the maximum revenue (i.e. using a Myerson's optimal mechanism with reserve prices) with n (symmetric) bidders is less than the revenue from an English auction with $n+1$ (symmetric) bidders. This result is striking because a Vickrey's auction is less demanding in terms of information. Indeed, to implement the optimal mechanism (in particular to find the reserve price) the seller should have more information about the distribution of the buyers' valuations than is required in a second price auction.[4] In the authors' words: "Therefore, under our assumptions, the seller should not accept any high "lock-up" bid that a buyer may be willing to offer in return for not holding an auction with an additional buyer."[5]

[1] The Top 14 rugby union brings together the fourteen best teams in France.

[2] This is also typically the case for public procurement where the buyer (a government or a public firm) asks for bids from several sellers.

[3] This under several assumptions. Among them: (i) risk neutrality, (ii) independence of buyers' private signals about the item's value, (iii) lack of collusion among buyers, (iv) no budget constraint. This optimal mechanism has a flavor of a second price auction with reserve prices: it allocates the item to the bidder who reports the highest virtual valuation.

[4] See also Krishna (2002) pages 73-75 or Milgrom (2004) 4.4.2 or Kirkegaard (2006).

[5] They also give the following real world example: "For example, in late 1993 Paramount agreed to sell itself to Viacom, knowing that QVC was interested in bidding for Paramount. Paramount and Viacom agreed to

The economist adviser should not keep it secret that the optimal auction characterized by Myerson requires one (or several if bidders are asymmetric) reserve price. That is a commitment from the seller not to sell if the maximum bid is below the reserve price.[6] This commitment can be difficult to make in practice. In the absence of a reserve price, however, the idea of setting an auction is even better. Indeed, with symmetric independent private-values the revenue equivalence theorem bites: all standard auctions[7] give the same expected revenue to the seller and they are also efficient (maximizing welfare).

Yet, in the business world, we observe many instances where a seller (of an indivisible good) does not use an auction. In fact, Aktas, de Bodt, and Roll (2010) study information from SEC filings between January 1,1994 and December 31, 2007. Among 1,774 transactions, they observe 847 one-on-one negotiations.[8] They find, however, that latent competition increases the bid premium offered in negotiated deals.

Recently, the FIFA (The Fédération Internationale de Football Association) sold 2026 World Cup television rights (for the USA) to Fox (who already had the rights for 2018 and 2022) without organizing a tender of offers.

We aim to see whether a buyer and a seller with antagonistic interests can find an agreement before an auction takes place, more specifically whether that have a common incentive to enter into exclusive pre-auction negotiations. Various papers have recently studied this type of situation. The literature started with Hua (2007) who . . . Choi (2009), Hua (2012).

Both a model and an experiment can be found in Grosskopf and Roth (2009).

From an empirical point of view, see Bajari, McMillan, and Tadelis (2009)

Our analysis applies for both selling or procurement. In the first context long-term relationship contract renewals repeated between a content right owner and broadcasters. or between a large food retailers and suppliers or public body and contractors contractual provisions that

we exclude resale in most of the analysis.

We study the anticompetitive effects of such negotiations in the spirit of Aghion and Bolton (1987): we show that is inefficient exclusion and that the incentives to collude come from rent-shift. One major difference is that there is an informational asymmetry between the colluding

terms that gave Viacom options to buy 24 million shares of Paramount and a $100 million break-up fee in the event that any other company were to purchase Paramount. The boards argued that in return for effectively excluding other bidders, Paramount had been able to negotiate a higher price than it could have expected in an open auction. QVC contested the terms of the deal, contending that holding an auction would have been the appropriate way to maximize shareholder value. The Delaware courts subsequently agreed with QVC. Our analysis supports that decision."

[6] See McAfee and Vincent (1997) for a model where the seller cannot commit organizing another auction if the good is not sold. They find that when the time between auctions goes to zero, the expected revenue converges to the one of a static auction with no reserve price.

[7] That is an auction which allocates the good to the highest bidder.

[8] some older references from takeovers

parties. Another one is that there is the natural outside option of running an auction. The focus is on the possibility of vertical collusion between the seller and one particular buyer.

We consider a seller facing potential buyers with symmetric valuations. The analysis applies in a symmetric way to a procurement context with one buyer facing potential suppliers with symmetric production costs. For clarity, we stick hereafter to the sale context.

Kirkegaard and Overgaard (2008) propose a model in which pre-auction offers (i.e. take it or leave it offers) can be profitable. They assume asymmetric bidders and that the seller is able to identify bidders'types. Hence the seller can approach bidders sequentially (from the highest type to the lowest) with an offer. If all offers are rejected, an auction starts. They study this game when the auction is a second price auction and when it is a first price one. Related on this literature on *asymmetric* auctions, a major difference is that in the present study the buyers are *symmetric* ex ante. Despite this symmetry, in equilibrium an asymmetric auction takes place with positive probability; the distribution of the types who participate in the auction is thus endogenous.

The analysis put forward a strong distinction between first-price and second-price auctions. The intuition with a first-price auction is as follows. In the first-price asymmetric auction, the weak buyer overbids and the strong buyer underbids. But as a final result the strong buyer bids too high and we show there is rent shifting. This does not happen in a second-price auction. This is a default of the first-price auction.

Also related to the literature on negotiations versus auctions Bulow and Klemperer (1996) Aktas, de Bodt, and Roll (2010). A difference is that we need a transfer ex ante to maximize joint surplus (as opposed to the maximization of solely the seller's profit). Difference in the timing: negotiation then auction

The article is organized as follows. We consider first a general signaling game that models exclusive pre-auction negotiations and gives rise to multiple equilibria. Then we use Cho and Kreps (1987)'s refinement criterion to rule out some of these equilibria. Next, we consider the equilibrium that maximizes the joint profit of the colluding parties. We explain the exclusionary and rent-shifting effects of each of the considered vertical agreements. Finally we examine a couple of extensions.

2 Model

A seller S sells an indivisible good to $n+1$ potential buyers $B_0, \cdots, B_n$. The seller and buyer B_0 have the opportunity to sign a contract before valuations are known, for instance because they have an ongoing business relationship to provide related products or services. Accordingly, we call B_0 the preferred buyer, and $B_1, \cdots, B_n$ the competing buyers. In our baseline model, the game proceeds as follows:

1. S sells the preferred buyer B_0 a purchase option with strike price $\bar{b}$;

2. The buyers $B_0, \cdots, B_n$ privately learn their valuations;

3. B_0 decides whether or not to exercise the option;

4. If the option is not exercised, a first-price auction with no reservation price takes place.

We assume that participation in the auction is costless, and consequently all buyers, including the preferred one, participate if the game gets to that point. The buyers' valuations are independent and identically distributed random variables. We denote by F their cumulative distribution function and assume that F admits a positive density f on $[0, 1]$.

How the competing buyers behave at the auction stage depend on what they know and understand of the preceding stages. Ex ante contracting may be held secretly and take place long before the time of the auction. Accordingly, a number of informational settings are possible. In a first scenario, the competing buyers are unaware of the existence of a preferred buyer and behave as if no action has taken place prior to the auction. For brevity, we say in this case that the competitors are "naive". In a second scenario, the existence of the ex ante contract as well as the value of the strike price are known to all players. In a third scenario, the competitors are aware of the existence of a purchase option but do not observe the strike price.

In each of these settings, the competing buyers determine their bids based on their beliefs about the distribution of the preferred buyer's valuations v_0 in the auction. For all possible competitors' beliefs, there exists a valuation threshold such that the auction takes place in equilibrium if and only if the preferred buyer's valuation is below the threshold.

Lemma 1. *For any strike price $\bar{b}$, there exists a unique valuation $\bar{v}$ such that the preferred buyer exercises the option if and only if his valuation v_0 is above $\bar{v}$. Conversely, for any cutoff valuation $\bar{v}$ in $[0, 1]$, there exists a unique strike price $\bar{b}$ that induces the preferred buyer to exercise the auction if and only if $v_0 \geq \bar{v}$.*

The preferred buyer with valuation $v_0 = \bar{v}$ is indifferent between exercising the option and participating in the auction. In the former case, he earns profit $\bar{v} - \bar{b}$ for sure. In the latter, he gets expected profit $(\bar{v} - b_0(\bar{v}))\bar{P}_0$, where $b_0(\bar{v})$ and $\bar{P}_0$ denote his bid and the probability that he wins the auction. The bid $b_0(\bar{v})$, the probability $\bar{P}_0$, and the expected profit $(\bar{v} - b_0(\bar{v}))\bar{P}_0$ depend on the competitors' beliefs about the distribution of the preferred buyer's valuation when the auction takes place. The specific relationship between $\bar{b}$ and $\bar{v}$ depends on those beliefs. But regardless of those beliefs, the relationship is increasing and the auction occurs when the preferred buyer's valuation for the good is low.

When $\bar{v} = 1$, the seller always resorts to competition to allocate the good, with all buyers participating in the symmetric first-price auction. On the contrary, when $\bar{v} = 0$, the seller and the preferred buyer operate under full exclusivity: competing buyers have no chance to purchase the good. Finally, when $\bar{v}$ lies in the interior of the support of the valuation distribution, we say that *partial exclusivity* prevails. The difference $1 - \bar{v}$ is a measure of the degree of exclusivity.

3 Naive competitors

We assume in this section that the competing buyers, should an auction take place, behave as if all bidders $B_0, \cdots, B_n$ were symmetric. Their behavior is thus described by the standard bidding function in the symmetric auction with $(n+1)$ players:

$$b^*_{n+1}(v) = \frac{1}{G_n(v)} \int_0^v x g_n(x) \mathrm{d}x = v - \frac{1}{G_n(v)} \int_0^v G_n(x) \mathrm{d}x, \tag{1}$$

where $G_n(v) = F(v)^n$ and $g_n(v) = nf(v)F(v)^{n-1}$ denote the cdf and pdf of the highest of n independent draws from F. The preferred buyer uses the bidding function b^*_{n+1} as well because this is the best reply to his competitors' strategy.

Given the strike price $\bar{b}$ and the competitors' belief that the support of v_0 is the whole interval $[0, 1]$, there exists a unique threshold $\bar{v}$ such that the auction takes place if and only if v_0 is below $\bar{v}$. When $v_0 \geq \bar{v}$, the preferred buyer exercises the option, deals with the seller regardless of the competitors' valuations, and earns profit $v_0 - \bar{b}_n$. When $v_0 \leq \bar{v}$, the auction takes place, the preferred buyer bids $b_0 = b^*_{n+1}(v_0)$ and wins the auction with probability $G_n(v_0)$. The indifference condition that determines the marginal type $v_0 = \bar{v}$ is therefore $\bar{v} - \bar{b} = \left(\bar{v} - b^*_{n+1}(\bar{v})\right) G_n(\bar{v})$. If $\bar{v} < 1$, the above equation, combined with (1), yields $\bar{b} = \bar{v} - \int_0^{\bar{v}} G_n(x)dx < b^*_{n+1}(\bar{v})$. Because the preferred buyer wins the auction with probability less than one, his bid has to be higher than the strike price to guarantee indifference.

At the ex ante contracting stage, the seller and preferred buyer choose the level of the strike price $\bar{b}$ that maximizes their joint expected profit, which they share using the price of the purchase option. As seen above, choosing the strike price is equivalent to choosing the marginal valuation $\bar{v}$ that makes the preferred buyer indifferent between exercising the purchase option and participating in the auction. Figures 1a an 1b show the corresponding allocation in the space (v_0, m), where $m = \max(v_1, \cdots, v_n)$ is the highest valuation for the good among the competing buyers.

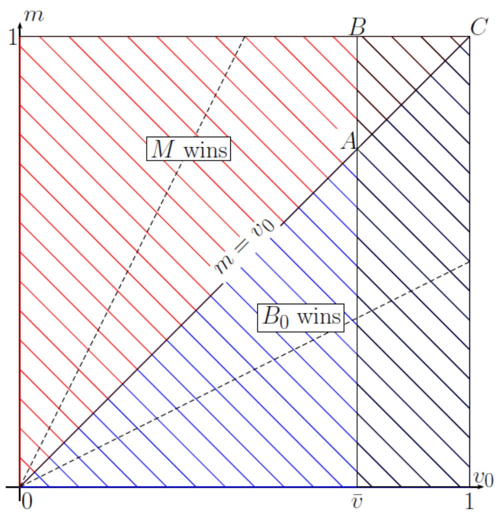


Figure 1a: Partial exclusivity with $b^*(1) < \bar{v} < 1$

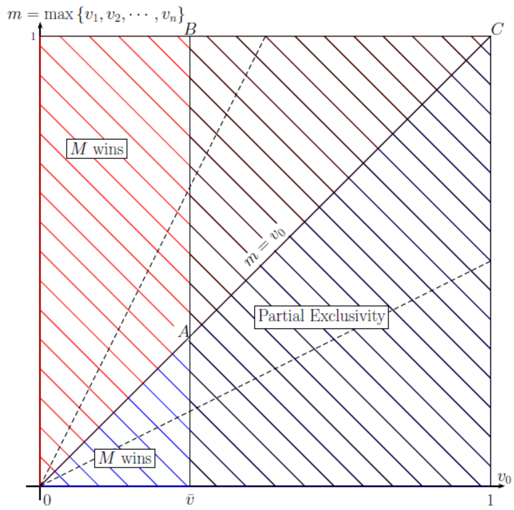


Figure 1b: Partial exclusivity with $\bar{v} < b^*(1)$

In the full competition regime, the first-price auction occurs with probability one, and the item is sold to buyer B_0 if and only if $v_0 \geq m = \max(v_1, \cdots, v_n)$. The allocation is the same under full competition ($\bar{v} = 1$) as under partial exclusivity ($0 < \bar{v} < 1$) except in the region $\bar{v} \leq v_0 \leq m \leq 1$ represented by the triangle ABC. In this region, under the partial exclusivity agreement, the coalition formed by the seller and the preferred buyer deals internally, earning joint profit v_0, while at least one competing buyer values the good more than the preferred buyer. Figures 1a shows the case where $\bar{v}$ is greater than the highest bid in the symmetric auction: $b^*(1) < \bar{v} < 1$. In this case, the coalition finds it optimal to forego the winning bid $b^*(m)$ because the surplus from internal trade is larger than the revenue from the auction: $v > b^*(1) \geq b^*(m)$ in the triangle ABC. As a result, partial exclusivity dominates full competition.

When there is one competing buyer ($n = 1$), the coalition earns the same profit under full exclusivity and under full competition because the bid placed by the competitor $b^*(v_1)$ coincides with the expectation of v_0 in the region $v_0 \leq v_1$ where the competitor wins the auction. For $n > 1$, the profit is strictly higher under full competition than under full exclusivity because the winning bid $b^*(m)$ increases with n. Partial exclusivity therefore dominates the two extreme regimes regardless of the number of competitors.

Proposition 1. *When the competing bidders are naive, partial exclusivity* ($0 < \bar{v} < 1$) *is optimal for the seller-preferred buyer coalition.*

Reducing $\bar{v}$ from $\bar{v} = 1$ to $\bar{v} = b^*(1)$, thereby increasing the degree of exclusivity in the relationship, unambiguously raises the coalition's profit because it expands the ABC area where the gain from internal trade dominates the highest bid received in the auction. Further reducing $\bar{v}$ has subtler effects, see Figure 1b: along the left boundary of the triangle (segment

AB), the highest bid $b^*(m)$ is greater than $\bar{v}$ for large values of m but lower than $\bar{v}$ for small values of m. We show in the appendix that the optimal degree of exclusivity $\bar{v}$ is such that the mean value of the difference $b^*(m) - v$ on the boundary AB is zero:

$$\int_{\bar{v}_n}^{1} \left[b^*_{n+1}(m) - \bar{v}_n\right] g_n(m)\mathrm{d}m = 0, \tag{2}$$

where g is the density function of $m = \max(v_1, \cdots, v_n)$. We also show that when both F and $1 - F$ are log-concave the above equation has a unique *interior* solution, which determines the degree of exclusivity in equilibrium. It follows that the coalition's profit has a unique interior global maximum and a local minimum at $\bar{v} = 1$.

Figure 2 shows the coalition's profit when the buyer valuations are uniformly distributed. When there is one single competitor, the maximal profit is obtained for $\bar{v} = 1/3$, which corresponds to the strike price $\bar{b} = 5/18$.[9] Table 2 line (e) shows the equilibrium profits of all players as well as the total welfare for the optimal value of $\bar{v}$. Compared to the first-price auction without reservation price, the coalition's profit increases by about 7.4%; the competing buyer's profit is more than halved and his probability of getting the good is almost halved, hence a strong exclusion effect. The welfare loss is about 7.4%.

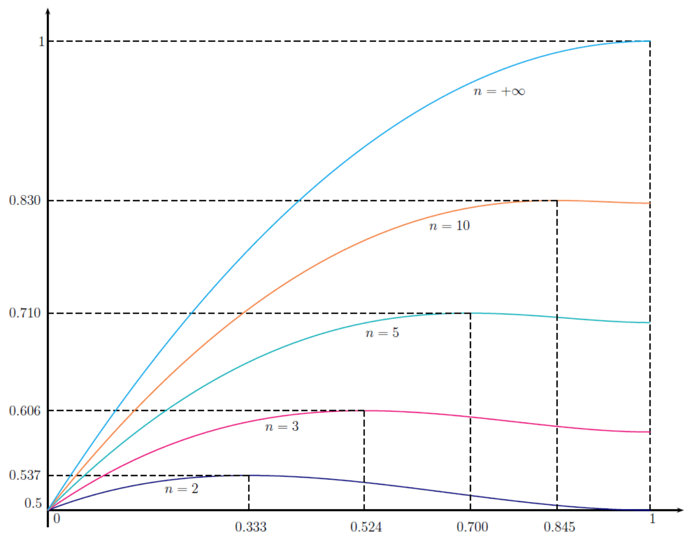


Figure 2: Coalition's profit under partial exclusivity (n competing buyers, uniform distribution)

Proposition 2. *Suppose that F and $1 - F$ are log-concave. The optimal degree of exclusivity decreases with the number of competitors. If $\tilde{F}$ stochastically dominates F according to the likelihood ratio order, there is less exclusivity for the valuation distribution $\tilde{F}$ than for the valuation distribution F.*

[9] As noticed above, the strike price is greater than the valuation placed by the indifferent preferred buyer should he participate in the auction, namely $b^*(\bar{v}) = 1/6$.

The comparative statics are proved in the appendix by using the characterization (2) of the optimal degree of exclusivity. When the number of competing buyers is higher or when valuations for the good are higher in the sense of the likelihood ratio order, the highest bid placed in the auction is higher, which makes resorting to the auction more attractive for the coalition. Moreover, the distribution of the competitors' highest valuation along the boundary AB places more weight on larger valuations, which further contributes to push $\bar{v}$ upwards, i.e., to decrease the optimal degree of exclusivity.

4 Sophisticated competitors

From now on, we assume that the contract signed ex ante by the seller and the preferred buyer is public. All players know the value of the strike price $\bar{b}$. The competing buyers understand that the valuations for the good are not symmetrically distributed among the bidders should the auction take place. The preferred buyer's valuation is drawn from the right-truncated distribution $F/F(\bar{v})$ with support $[0, \bar{v}]$, which we denote by F_0. In the terminology of Maskin and Riley (2000), the preferred buyer is a "weak bidder" and the competing buyers are "strong bidders" in the asymmetric auction.

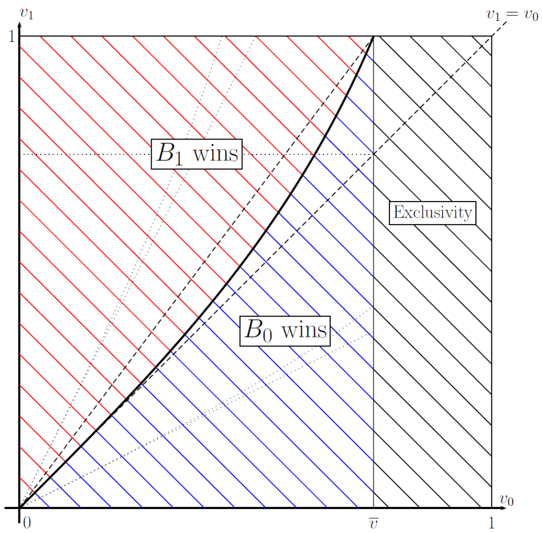


Figure 3: Partial exclusivity with sophisticated buyers

We start with the case where there is only one competing buyer. Figure 3 shows the equilibrium allocation with partial exclusivity $0 < \bar{v} < 1$. In the exclusivity region $v \geq \bar{v}$, the preferred buyer exercises the purchase option. For $v \leq \bar{v}$, the asymmetric auction takes place. We denote by b_0 and b_1 the bidding functions and by ϕ_0 and ϕ_1 the inverse bidding functions of the two bidders. The bidding functions b_0 and b_1 have the same support, with upper bound $b_0(\bar{v}) = b_1(1)$. The bold line with equation $v_0 = \phi_0(b_1(v_1))$ separates the regions where each player wins the auction.

The preferred buyer with type $v_0 = \bar{v}$ is indifferent between participating in the auction and exercising the purchase option. In the first case, he wins the auction with probability one and hence earns $\bar{v} - b_0(\bar{v})$; in the latter case, he earns $v_0 - \bar{b}$. The indifference condition is therefore $\bar{b} = b_0(\bar{v})$. Hence, contrary to the naive competitor case, the strike price and the maximal bid placed by the preferred buyer in the auction coincide. The preferred buyer strongly prefers participating in the auction rather than exercising the purchase option when his valuation is below $\bar{b}$. When his valuation is above that threshold, he is indifferent and we assume that he exercises the option.

Proposition 3. *Suppose there is one competing buyer and the competitor is sophisticated. Then the joint profit of the seller and the preferred buyer is strictly higher under partial exclusivity ($0 < \bar{v} < 1$) than under full exclusivity ($\bar{v} = 0$) or full competition ($\bar{v} = 1$).*

As already noticed, the coalition's profit is the same under full exclusivity and full competition when there is only one competing buyer. This is because the bid $b^*(v_1)$ placed by the competitor in the symmetric auction coincides with the expectation of the preferred buyer's valuation v_0 in the region where the competitor wins the auction. It is therefore enough to compare the partial exclusivity and full exclusivity regimes. To this aim, we show in the appendix that for any v_1 the bid $b_1(v_1)$ placed by the competitor in the asymmetric auction is *higher* than the expectation of v_0 in the region where the competitor wins the asymmetric auction, namely $0 \leq v_0 \leq \phi_0(b_1(v_1))$. Specifically, we show that $b_1(v_1)$ is equal to the expectation of v_0 on a *larger* set of values for v_0. (This larger set corresponds to the region where the competitor would win if the preferred buyer were to bid less aggressively than he actually does, see the proof in the Appendix for details.)

We conclude from the above analysis that even though the competing buyer rationally lowers his bid knowing that he faces a weak bidder, it remains more profitable for the coalition to run the auction when the preferred buyer's valuation is low than to maintain a completely exclusive relationship. For instance, when the buyer valuations are uniformly distributed on $[0, 1]$, the maximum bid in the asymmetric auction is $\bar{b} = \bar{v}/(1 + \bar{v})$ and the inverse bidding functions are given by

$$\phi_0(b) = \frac{2b}{1 + z\left(b/\bar{b}\right)^2} \quad \text{and} \quad \phi_1(b) = \frac{2b}{1 - z\left(b/\bar{b}\right)^2}, \tag{3}$$

with $z = 1 - 2\bar{b}$, see Maskin and Riley (2000). The bidding functions are represented on Figure 3, PANEL A, à faire. Numerical computations show that the coalition's profit has a unique maximum, which is obtained for $\bar{v}$ approximately equal to .505. The corresponding strike price is $\bar{b} = \bar{v}/(1 + \bar{v}) = .34$. Thus, the degree of exclusivity is lower than in the naive competitor case (recall that $\bar{v} = 1/3$ in that case). Table 2 line (f) shows the equilibrium profits of all players as well as the total welfare for the optimal value of $\bar{v}$. Compared to the first-price auction, the coalition's profit gain from the agreement is 2.75%, instead of 7.4% when the competitor is naive. The welfare loss is only 3.75%. Because the competing buyer understands that he should reduce his bid in the auction, his expected profit is 25% lower than in the first-price auction, instead of more than 50% when he is naive. In sum, compared to the naive competitor case, the exclusion effect and the competitive harm are still present, but less pronounced.

The above results heavily rely on the presence of a single competing buyer. When there is more than one competitor, resorting to competition is more attractive for the coalition because the revenue earned from the auction is likely to be higher. Yet, as stated in the proposition below, partial exclusivity continues to prevail, at least with uniformly distributed buyer valuations.

Proposition 4. *When the valuations are uniformly distributed, partial exclusivity ($0 < \bar{v} < 1$) is optimal for the seller-preferred buyer coalition regardless of the number of competitors.*

As already noticed, the coalition prefers full competition ($\bar{v} = 1$) to full exclusivity ($\bar{v} = 0$) when there are two or more competing buyers. It is therefore enough to compare the full competition and partial exclusivity regimes. We do so by slightly perturbing the full competition regime, i.e., by slightly reducing $\bar{v}$ below one. Such a perturbation leads the sophisticated competing buyers to slightly reduce their bids and the preferred buyer to slightly raise his bid compared to the symmetric auction. Each of these two forces push the probability that the coalition deals internally upwards.

The increased probability of an internal deal is associated with a positive effect on the coalition's profit because at the margin it earns $v_0 = m$ instead of the winning bid $nm/(n+1)$, where $m = \max(v_1, \cdots, v_n)$ is the highest valuation among the competitors. On the other hand, when the competitors win the auction, the collected revenue is reduced because their bids are lower. We show in the appendix that the second force mentioned above (the preferred buyer slightly raising his bid) is enough to offset the negative revenue effect. Hence the total net effect of introducing some exclusivity in the coalition's relationship is positive. Whether the result holds for more general valuation distributions than the uniform distribution is left as an open problem.

5 Unobservable strike price

We now assume that the competing buyers know that a purchase option has been granted ex ante, but do not know the value of the strike price. Sophisticated competitors therefore have to form a belief about the distribution of the preferred buyer's valuations when the auction takes place. By Lemma 1, it is known to all players that the preferred buyer participates in the auction if and only if his valuation is below a certain threshold. In other words, the distribution of v_0 given that the auction takes place is necessarily a right-truncation of the original distribution F. As a result, the competitors' belief can by summarized by an expected cut-off valuation $\bar{v}^e$.

Suppose first there is only one competitor and the competitor expects that the preferred buyer participates in the auction when v_0 is lower than or equal to $\bar{v}^e$. The competitor therefore bids as in the asymmetric auction where the weak bidder has his valuation drawn from the right-truncated distribution $F/F(\bar{v}^e)$ on $[0, \bar{v}^e]$. Let $\bar{b}^e = b_0(\bar{v}_e) = b_1(1) < \bar{v}^e$ be the upper bound of the common distribution of the two players' bids in this auction. As explained in Section 4, if the coalition ex ante agrees on the strike price $\bar{b}^e$, the preferred buyer strongly prefers participating in the auction rather than exercising the purchase option when his valuation is below $\bar{v}^e$. When his valuation is above that threshold, he is indifferent and we assume that he uses the option.

Suppose now that the coalition agrees instead on a strike price $\bar{b}$ that is slightly below $\bar{b}^e$. Then the preferred buyer participates in the auction when his valuation is below $\bar{v}$, where $\bar{v}$ is slightly below $\bar{v}^e$. The only change in the allocation occurs for $\bar{v} \leq v_0 \leq \bar{v}^e$ and values v_1 high enough for the competitor to win the auction at the candidate equilibrium: in this region, represented by the DEF area on Figure ???, the coalition deals internally after the deviation, thus earning $\bar{v}^e$ instead of $\bar{b}^e < \bar{v}^e$. The coalition therefore has an incentive to slightly increase the degree of exclusivity compared to what was expected by the competitor. A standard unraveling argument shows that the only possible equilibrium is $\bar{v}^e = \bar{v} = 0$, full exclusivity.

FIGURE SHOWING THE UNRAVELLING PROCESS

Proposition 5. *Suppose the competing buyers are sophisticated, know the existence of the purchase option but ignore the value of the strike price $\bar{b}$.*

Then there is a unique Bayesian equilibrium, which is characterized by the exclusivity threshold $\bar{v}^e_n$. When there is one competitor, full exclusivity prevails ($\bar{v}^e_n = 0$). When there are two competitors or more, partial exclusivity prevails, and we have

$$\bar{v}_{n-1} \leq \bar{v}^e_n \leq b^*_n(1),$$

where $\bar{v}_{n-1}$ is the optimal exclusivity threshold in the presence of $n-1$ naive competitors.

The unraveling argument exposed above heavily relies on the fact that the distributions of bids have the same support for the preferred and competing buyers, and more specifically on the implication that the highest bid placed by the competitor is lower than $\bar{v}$. This is not necessarily true, however, when there are two (or more) competitors. Indeed the strong bidders behave at least as aggressively as if the weak bidder were absent. As a result, when the number of competitors is high and $\bar{v}$ is low, their highest bids may be at levels that the weak bidder is unable to reach. In this case, when the competitors' valuations are high, they actually behave as in the absence of the weak bidder, using the bidding function b_n^*. In particular, the highest bid placed by the competitors is $b_n^*(1)$. When $\bar{v}^e$ is below that value, the unraveling argument, such as exposed above, does no longer apply: resorting competition becomes more attractive to the coalition.

In equilibrium, the expected and actual degrees of exclusion coincide, $\bar{v}_n^e = \bar{v}_n$, and satisfy the first-order condition:

$$\int_{\phi_n^*(b_0(\bar{v}_n^e))}^{1} \left[b_n^*(m) - \bar{v}_n^e\right] g_n(m)\mathrm{d}m = 0 \tag{4}$$

for $n \geq 2$.

Since B_0 bids more aggressively than in the symmetric auction: $b_0(v) \geq b_{n+1}^*(v) \geq b_n^*(v)$, we have $\phi_n^*(b_0(\bar{v}_n^e)) \geq \bar{v}_n^e$ and from (4) we get

$$0 \geq \int_{\bar{v}_n^e}^{1} \left[b_n^*(m) - \bar{v}_n^e\right] g_n(m)\mathrm{d}m \geq \int_{\bar{v}_n^e}^{1} \left[b_n^*(m) - \bar{v}_n^e\right] g_{n-1}(m)\mathrm{d}m,$$

which yields $\bar{v}_n^e > \bar{v}_{n-1}$ by (2).

6 Priority right

Second, we relax the assumptions on the form of the ex ante contract. consider a variant of the game where at stage 1 the seller does not sell the preferred buyer a purchase option, but merely the right to make an offer at stage 3 before the auction takes place.

We consider the possibility that the seller and a particular buyer ex ante sign a contract that allows them to enter into exclusive negotiations after the uncertainty is resolved. Exclusive negotiations take place under asymmetric information and are modeled with a take-it-or-leave-it offer from the informed party, namely the buyer. The negotiation therefore gives rise to a signaling game.

1. S sells I the right to enter into exclusive negotiations after he has privately learnt his valuation;

2. E and I privately learn their valuation;

3. Under exclusive negotiations, I makes a take-it-or-leave-it offer, $p(v_I)$;

4. If S rejects the offer, the good is sold in a first-price auction with no reservation price.

The other player, buyer E, is aware that exclusive negotiations have taken place prior to the auction, but does not observe the content of these negotiations.

Proposition 6. *For all $\bar{v}_I \in [0,1]$ and $\bar{b} = \bar{v}_I/(1+\bar{v}_I) \in [0,1/2]$, the following configuration is an equilibrium of the signaling game (stages 3 and 4):*

- *At the negotiation stage, the buyer offers $p(v_I) = \bar{b}$ if $v_I > \bar{v}_I$ and $p(v_I) = 0$ otherwise;*
- *S accepts to sell at $\bar{b}$ and rejects the zero price offer;*
- *When the offer is rejected, an asymmetric auction takes place with v_E and v_I being independently distributed on $[0,1] \times [0,\bar{v}_I]$;*
- *If the seller receives an out-of-equilibrium offer, she believes that $v_I \geq \bar{v}_I$.*

Proof. The asymmetric auction at step 4 has been studied by Maskin and Riley (2000). The two players bid in the same range $[0,\bar{b}]$ and the equilibrium inverse bidding functions $\phi_E = (b_E)^{-1}$ and $\phi_I = (b_I)^{-1}$ are given by

$$\Phi_E(b) = \frac{2b}{1 - z\left(b/\bar{b}\right)^2} \quad \text{and} \quad \Phi_I(b) = \frac{2b}{1 + z\left(b/\bar{b}\right)^2}, \tag{5}$$

with $z = 1 - 2\bar{b}$. Conditionally on v_I, the probability that I wins the auction is

$$\mathbb{P}\left(I \text{ wins} \,|\, v_I\right) = \mathbb{P}\left(b_E(v_E) \leq b_I(v_I)\right) = \Phi_E(b_I(v_I)),$$

which is increasing and convex in v_I because the two functions Φ_E and b_I are increasing and convex. The incumbent buyer's expected gain is

$$U_I(v_I) = [v_I - b_I(v_I)]\, \Phi_E(b_I(v_I)), \tag{6}$$

which is increasing and convex in v_I with slope $U_I'(v_I) = \Phi_E(b_I(v_I))$. We extend the function b_I on the whole interval $[0,1]$ by setting $b_I(v_I) = \bar{b}$ on $[\bar{v}_I, 1]$. A buyer of type $v_I > \bar{v}_I$, should he participate in the auction, would indeed bid $\bar{b}$ and get the good with certainty.

Observing buyer I's offer $\bar{b}$, the seller believes that $v_I \geq \bar{v}_I$ and is indifferent between accepting and rejecting the offer. She obviously rejects the zero offer and would accept any out-of-equilibrium offer above $\bar{b}$. Finally if she received an offer lying between zero and $\bar{b}$, she would reject it under the out-of-equilibrium belief stated in the Proposition because she would expect to earn $\bar{b}$ at the auction.

Consider now the choice of buyer I's take-it-or-leave-it offer at step 3. If his type is equal or above $\bar{v}_I$, he is indifferent between any offer equal or below $\bar{b}$ and does not want to make an offer above that level. If his type is below $\bar{v}_I$, he is indifferent between any offer strictly below $\bar{b}$ (as all such offers are rejected) and does not want to offer $\bar{b}$ (or a fortiori any price higher than $\bar{b}$) because $U_I(v_I) > v_I - \bar{b}$. □

7 Discussion

Corollary 1. *When the distribution of valuation is uniform, the optimal cut-off is given by*

$$\bar{v} = \left(\frac{n}{n+1}\right)^2 + \left[1 - \left(\frac{n}{n+1}\right)^2\right] (\bar{v})^{n+1}$$

Proof. Assume $F(v) = v$ then $G(v) = v^n$, $g(v) = nv^{n-1}$, and $b^*(v) = \frac{n}{n+1}v$. Substituting these expressions into the f.o.c. it writes

$$-\bar{v}\left(1 - \bar{v}^n\right) + \frac{n^2}{n+1}\int_{\bar{v}}^{1} v^n dv = 0$$

or

$$\bar{v} - \bar{v}^{n+1} = \left(\frac{n}{n+1}\right)^2 \left[1 - \bar{v}^{n+1}\right]$$

and finally

$$\bar{v} = \left(\frac{n}{n+1}\right)^2 + \left[1 - \left(\frac{n}{n+1}\right)^2\right] (\bar{v})^{n+1}$$

□

7.1 Optimal mechanism

We have considered here a must-sell auction. Can we say something if we allow the good not to be sold (auction with reservation price)?
Link with vertical integration? Same as allowing for resale? Take-it-or-leave-it offer to E at price $(1 + v_I)/2$?

Implementable by reselling or reservation price.

Before turning to our timing, a useful benchmark is the AB model slightly adapted to fit with our environment.[10] In AB there are three players. Ex ante, the seller S and the B_0 can both observe v_0 but not the valuation of the other buyer B_1. Then S and the B_0 can use their knowledge of the value v_0 to extract rents from B_1. They commit on two prices: $p_0(v_0)$ and $p_1(v_0)$. Ex post, B_1 observes its valuation v_1 and if B_1 agrees to pay the price, p_1, he obtains the good, otherwise the good is allocated to B_0 for the price p_0. The price p_1 is used to extract rents from B_1 and the price p_0 to share these rents between S and B_0.

Given p_1, B_1 buys if and only if $v_1 > p_1$. Therefore the joint profit of S and B_0 writes:

$$\left(1 - F(p_1)\right) p_1 + F(p_1) v_0 = v_0 + \left(1 - F(p_1)\right)\left(p_1 - v_0\right)$$

[10] The main difference, which is purely formal, is that in AB there a buyer and two sellers whereas here we have a seller and two buyers.

The pair $S - B_0$ maximizes, thus, the monopoly profit of a firm facing a demand $1 - F(p_1)$ and a constant marginal cost v_0. That is, p_1 is solution to

$$p_1 = v_0 + \frac{1 - F(p_1)}{f(p_1)} \tag{7}$$

The consequences are twofold. First, B_1 pays more than in an auction (rent shifting). Indeed, AB assumes that without an agreement between S and B_0 the good is sold through a second price auction (i.e. Bertrand competition in their procurement setting). In such an auction, when $v_1 > v_0$, B_1 obtains the good and pays v_0. Otherwise the good is sold to B_0 and B_1 pays zero. As $p_1(v_0)$ is the monopoly price for a marginal cost of v_0 it is larger than v_0. Second, whenever $v_0 < v_1$ but $v_1 < p_1(v_0)$, B_1 does not buy when the AB contract is in place (exclusion). This exclusion occurs with probability $F(p_1(v_0)) - F(v_0)$.

This analysis extends to more buyers: $B_1, \ldots, B_n$, $n \geq 2$. The valuations of the buyers being iid and private values. Ex ante, the pair $S - B_0$ commits on two prices $p_0(v_0)$ and $p_1(v_0)$. Ex post, the buyers $B_1, \ldots, B_n$ privately observe their valuations and participate in a second price auction where $p_1(v_0)$ is the public reserve price.[11] In an auction (either first or second price) with a reserve price the reserve price matters only when there is one bidder with a valuation above it while all the others have a valuation below the reserve price. Hence, the optimal reserve price does not depend on n and the joint profit of S and B_0 is still maximized by p_1 given by (7).[12]

In our context, the natural auction mechanism is a first price auction. This is a first departure from AB. However, substituting the second-price auction by a first-price auction in AB model does not change their results. Indeed, it is easy to see that B_1 would pay less in an auction recall that in an auction with two bidders B_1 bids $b(v_1) = \mathbb{E}\left[v \,|v < v_1\right]$ and wins whenever $v_1 > v_0$. Exemple: $F(.)$ uniform, then $b(v_1) = v_1/2 < 1/2 < 1/2 + v_0/2 = p_1(v_0)$. In addition, B_1 is (inefficiently) prevented from buying when $v_0 < v_1 < (1 + v_0)/2$.

idea plan: prop-5 de Burguet-Perry (section 4) + lien avec AB

mais AG demande un "gros" pouvoir d'engagement...

literature part sur rofr mais aussi anti-concu...

idea autre benchmark: S et I negocient mais si echec, alors I est out (similar to naive case ?)

7.2 Right of first refusal

Comparison: ROFR perception effect –> change in bid can be in any direction. Or no change in the uniform case.

[11] Comments in the following spirit: In an environment where the seller cannot credibly commit not to sell, the ex-ante agreement between the seller and B_0 makes the reserve price more credible.

[12] For technical details, see Krishna (2002) pages 24-26.

While for us, sophistication induces under reasonnable assumption induces underbidding by competitors because they understand they face a weak bidder.

Walker (1999) is (probably) the article which started a flow of models on the ROFR. It presents a (mostly descriptive) law and economics perspective on the subject and rather argue against the ROFR.

Bikhchandani, Lippman, and Ryan (2005) put forward negative consequences for the seller of granting a ROFR (affiliated values, $N \geq 2$). Indeed, such a right increases the profit of the favored buyer but they show that "usually" it decreases the joint profit of the seller and this favored buyer. Therefore the favored buyer "usually" cannot compensate the seller. Arozamena and Weinschelbaum (2006) similarly show that ROFR cannot be part of an optimal mechanism (in the ipv model) (what do they show in Arozamena and Weinschelbaum (2009b)?). But it does not mean it cannot increase the profit of the seller. Chouinard (2005) models the ROFR in the context of a first-price auction (with a reserve price). He finds that the seller loses when the ROFR is present. His result comes from the absence of ex-ante transfer between the right holder and the seller.

Hua (2007) (and most the remaining of the literature) focuses on $n = 2$. He characterizes the mechanism which maximizes the seller profit when the favored buyer has a limited budget. Otherwise if the favored buyer had unlimited budget, the seller would ex ante sell the good to the favored buyer who could resell it to the other buyer ex-post (see footnote 8). It is amusing that the optimal (unconstrained) mechanism is relegated to a footnote!

The ROFR clause is further examined in Choi (2009) and Burguet and Perry (2009). In both papers, the favored buyer has no budget constraint. Choi (2009) has two bidders but he consider affiliated values. Contrary to Bikhchandani, Lippman, and Ryan (2005) he focuses on first-price aution. He shows that the ROFR increases the joint profit of the seller and the favored buyer. Burguet and Perry (2009) allows for $N \geq 2$ in a setting similar to Choi. In both papers, the equilibrium bids of the regular bidders can be hard to find (they are characterized by a differential equation which might be not obvious). Finally, Burguet and Perry show how the seller could extract the value of v_1 in a mechanism and generate the maximum profit (using a first-price auction with a reserve price adjusted for each value of v_1).

Also Arozamena and Weinschelbaum (2009a) (corruption)

Grosskopf and Roth (2009) model the ROFR in the context of an Ultimatum game (and a reverse Ultimatum game) and they run an experiment. See Brisset, Cochard, and Maréchal (2015) for another experiment.

Common assumptions: risk-neutrality, seller's value is zero, no entry cost, (technical assumptions on hazard rate), Welfare comparison depends on the chosen reference (e.g. auction with or without a reserve price in the absence of an ex-ante agreement).

Payoffs comparisons in the case of the uniform distribution and $n = 2$.

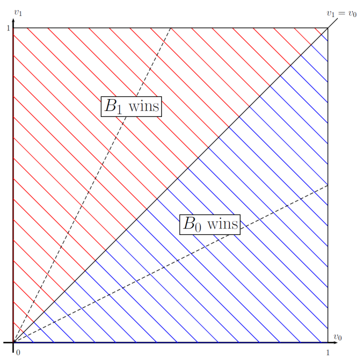
Table 1: Main assumptions in ROFR papers

	Must sell	nb of Buyers	Information	Auction	$\Pi_I + \Pi_S$ (vs no right)	what else ?
BLR, 2005	yes	$n+1$	APV	2nd price	$\searrow$ (mostly) or $\nearrow$	
Chouinard, 2005	no	2	IPV	1st price	$\Pi_S \searrow$	
Hua, 2007	no	2	IPV	mechanism	$\nearrow$	
Choi, 2009	yes	2	APV	1st or 2nd	$\Pi_I + \Pi_S \nearrow$	
BP, 2009	yes & no	$n+1$	IPV	1st & var	$\Pi_I + \Pi_S \nearrow$	
AW, 2009	yes	n	IPV	1st price	$\Pi_I + \Pi_S \nearrow$	

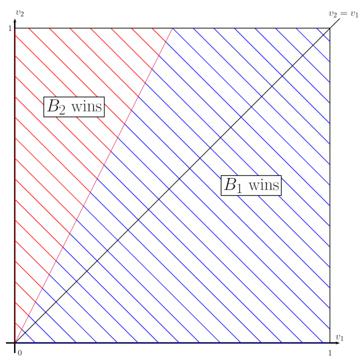
Table 2: Equilibrium allocations under various mechanisms

	$\Pi_0^\dagger$	$\Pi_S^\dagger$	$\Pi_0 + \Pi_S^\dagger$	$\Pi_1^\dagger$	$W^\dagger$	$\Pr(B_0 \text{ wins})$	$\Pr(B_1 \text{ wins})$
(a) 1st price	4	8	12	4	16	.5	.5
(b) 1st price with res. price .5	2	10	12	2	14	.325	.325
(c) Optimal mechanism	n/a	n/a	14	1	15	.75	.25
(d) Right of first refusal	7	6	13	2	15	.75	.25
(e) Purchase option (Naive)*	5.48	7.40	12.89	1.93	14.81	.72	.28
(f) Purchase option (Sophist.)*	5.54	6.8	12.33	3	15.4	.66	.34

[1] There is one competing buyer ($n = 1$). Buyer valuations are uniformly distributed.
[2] Π_0, Π_1, and Π_S are the preferred buyer's, competing buyer's, and seller's ex post expected profits, without accounting for ex ante transfers between S and B_0.
[3] Total welfare W is the sum $\Pi_0 + \Pi_1 + \Pi_S$.
[4] (†) All figures in column are to be divided by 24.
[5] (*) All figures in line are approximated values.



Absolute FP auction



FP auction + ROFR

The role of rent shifting The ex ante contract between the seller and I has the flavor of a vertical integration. At first, it seems counter intuitive for S to deal exclusively with I instead of going directly to an auction (and making I and E compete). Yet, by dealing together S and I can increase their joint surplus. In a symmetric auction, I would win when $v_I > v_E$. When I loses S deals with E and obtain only part of E surplus.

To illustrate, in a *first price* auction when $v_I < v_E$, E would pay $\mathbb{E}\left[v|v < v_E\right]$ (i.e. $v_E/2$ with a uniform distribution). Now, if $\mathbb{E}\left[v|v < v_E\right] < v_I < v_E$ (i.e. $v_E/2 < v_I < v_E$ with a

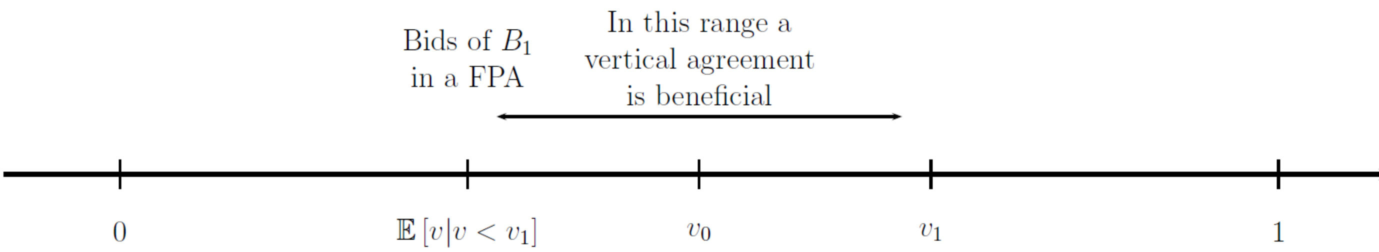


Figure 4: Time line

uniform distribution), S and I are better off in an exclusive deal than if S sells to the more efficient buyer.

Notice that with a second price auction, the exclusive deal does not improve the joint surplus of S and I as whenever $v_I < v_E$, E pays v_I (in equilibrium when dominated strategy are discarded) anyway. But when the selling mechanism is (akin to) a first price auction, then an exclusive deal can improve the joint profit of S and I.

Ideally, the exclusive deal should apply when v_I is large enough (i.e. $\mathbb{E}[v'|v' < v_E] < v_I$) and not apply when v_I is low. As v_E is not observe this ideal cutoff cannot be implemented. Thus a tradeoff.

7.3 Contracting with many buyers

2 buyers at the negotiation stage

voire k parmi n !

Renegotiation proofness Dans le cas ou bbar observable par E

7.4 Second-price auction

If one competitor, n=1, in the baseline model, coalition payoff independent of $\bar{b}$. If $n > 1$, partial exclusivity is suboptimal.

same for ROFR

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APPENDIX

A Appendix

Proof of Lemma 1 If the auction takes place, the competing buyers' bidding functions $b_1, \ldots, b_n$ depend on their beliefs about the distribution of the preferred buyer's valuations. The competitors' beliefs and bidding strategies are taken as given in equilibrium. (We describe them later for each informational scenario that we consider.) The preferred buyer B_0 with type v_0 earns expected profit

$$\Pi_0(v_0) = \max_b (v_0 - b) \Pr\left[b \geq \max(b_1(v_1), \ldots, b_n(v_n))\right].$$

By the envelope theorem, Π_0 increases with v_0, with the derivative $\Pi_0'(v_0)$ being equal to the probability that B_0 wins the auction. Moreover, the expected profit Π_0 is convex in v_0 because it is the upper bound of a family of affine functions. The probability that B_0 wins the auction is therefore nondecreasing in v_0.

On the other hand, the gain from exercising the purchase option is $v_0 - \bar{b}$. The preferred buyer prefers exercising the option rather than participating in the auction if and only if $v_0 - \bar{b} \geq \Pi_0(v_0)$. As $\Pi_0' \leq 1$ on $[0, 1]$, the function $v_0 - \Pi_0(v_0)$ is nondecreasing on this interval, and therefore the inequality $\bar{b} \leq v_0 - \Pi_0(v_0)$ holds if and only if v_0 is greater than or equal to a threshold that we denote by $\bar{v}$. This threshold is nondecreasing in the strike price $\bar{b}$. By convention, we may set $\bar{v}$ to one when the inequality never holds.

It may be the case that for large values of v_0 the preferred buyer wins the auction with probability one and is indifferent between participating in the auction and exercising the purchase option. (In such a situation, $v_0 - \bar{b}$ and $\Pi_0(v_0)$ coincide on an interval that contains $v_0 = 1$). To avoid uninteresting complications, we assume that in case of indifference, the preferred buyer exercises the option.

Proof of Proposition 1 Under partial exclusivity ($0 < \bar{v} < 1$), the joint profit of the SB_0 coalition is:

$$\Pi_S(\bar{v}) + \Pi_0(\bar{v}) = \int_{\bar{v}}^1 v f(v) \mathrm{d}v + \int_0^{\bar{v}} vG(v) f(v) \mathrm{d}v + \int_0^{\bar{v}} \int_v^1 b^*(x) g(x) \mathrm{d}x \, f(v) \mathrm{d}v,$$

where the last term corresponds to the case where the auction takes place and is won by the competing buyer with the highest valuation for the good. The first and second terms correspond to the cases where the good is allocated to the preferred buyer and the coalition joint profit is v. Under full competition ($\bar{v} = 1$), there is no exclusivity arrangement and the first term disappears:

$$\Pi_S(1) + \Pi_0(1) = \int_0^1 vG(v) f(v) \mathrm{d}v + \int_0^1 \int_v^1 b^*(x) g(x) \mathrm{d}x \, f(v) \mathrm{d}v.$$

By difference, the terms v or b^* cancel out except in the region $\bar{v} \le v \le x \le 1$ represented by the triangle ABC on Figures 1a and 1b:

$$\Pi_S(\bar{v}) + \Pi_0(\bar{v}) - [\Pi_S(1) + \Pi_0(1)] = \int_{\bar{v}}^1 \int_v^1 [v - b^*(x)]\, g(x)\mathrm{d}x\, f(v)\mathrm{d}v. \tag{A.1}$$

The above difference is positive when $\bar{v} > b^*(1)$ as $v - b^*(x) \ge \bar{v} - b^*(1)$ in the triangle. It follows that partial exclusivity, with $b^*(1) < \bar{v} < 1$, is preferred to full competition ($\bar{v} = 1$), which itself, as explained in the main text, is weakly preferred to full exclusivity ($\bar{v} = 0$) by the coalition.

Differentiating (A.1) with respect to $\bar{v}$, we find that the first derivative of the joint profit is

$$\Pi_S'(\bar{v}) + \Pi_0'(\bar{v}) = \int_{\bar{v}}^1 [b^*(m) - \bar{v}]\, g(m)\mathrm{d}m. \tag{A.2}$$

The degree of exclusivity chosen by the coalition, which is known to be at the interior of the support of the valuation distribution, therefore satisfies the first-order condition (2).

Proof of Proposition 2 We first show that if G and $1-G$ are log-concave the coalition's profit is quasi-concave in $\bar{v}$. The second derivative of the joint profit is given by:

$$\Pi_S''(\bar{v}) + \Pi_0''(\bar{v}) = g(\bar{v}) \left[\bar{v} - b^*(\bar{v}) - \frac{1 - G(\bar{v})}{g(\bar{v})}\right].$$

If F is log-concave, so is $G = F^n$, and so is $\int_0^v G(x)\mathrm{d}x$ by Theorem 1 in Bagnoli and Bergstrom (2005). It then follows from (1) that $\bar{v} - b^*(\bar{v})$ increases in $\bar{v}$. Since, by assumption $(1-G)/g$ is decreasing, the bracketed term is increasing in $\bar{v}$. Because that term is negative at $\bar{v} = 0$ and positive at $\bar{v} = 1$, the coalition's profit is first concave then convex as $\bar{v}$ rises from zero to one. We already know that the coalition's profit has an interior global maximum and a local minimum at $\bar{v} = 1$. We can therefore conclude that the interior maximum is unique. The profit derivative (A.2) is first positive then negative, with a unique zero in the interior of the support of the valuation distribution, which characterizes the optimal degree of exclusivity.

We now compare the equilibria with n and $n+1$ competitors. We denote by $\bar{v}_n$ and $\bar{v}_{n+1}$ the optimal degrees of exclusivity and by b_n^* and b_{n+1}^* the bidding functions in the two situations. We set $G_n = F^n$ and $G_{n+1} = F^{n+1}$. For any $\bar{v}$, the left-truncated distribution $G_{n+1}/(1 - G_{n+1}(\bar{v}))$ on $(\bar{v}, 1)$ first-order stochastically dominates the truncated distribution $G_n/(1 - G_n(\bar{v}))$ on the same interval.[13] We have

$$\begin{aligned} 0 = \int_{\bar{v}_n}^1 [b_n^*(m) - \bar{v}_n] \frac{\mathrm{d}G_n(m)}{1 - G_n(\bar{v}_n)} \quad &\le \quad \int_{\bar{v}_n}^1 [b_{n+1}^*(m) - \bar{v}_n] \frac{\mathrm{d}G_n(m)}{1 - G_n(\bar{v}_n)} \\ &\le \quad \int_{\bar{v}_n}^1 [b_{n+1}^*(m) - \bar{v}_n] \frac{\mathrm{d}G_{n+1}(m)}{1 - G_{n+1}(\bar{v}_n)}. \end{aligned}$$

[13] Algebraic computations show that $b^n - a^n/(1-a^n)$ decreases with n for $0 \le a \le b \le 1$. Applying this property with $a = F(\bar{v})$ and $b = F(v)$ for $\bar{v} \le v \le 1$ shows the desired property.

The first equality is the first-order condition when there are n competing buyers. The first inequality follows from $b^*_{n+1} \geq b^*_n$. The second inequality follows from the monotonicity of the function b^*_{n+1} and the first-order stochastic ordering of the distributions $G_{n+1}/(1-G_{n+1}(\bar{v}_n))$ and $G_n/(1-G_n(\bar{v}_n))$ on the interval $(\bar{v}_n, 1)$. Because the problem is quasi-concave in $\bar{v}$, we conclude that $\bar{v}_{n+1} \geq \bar{v}_n$: the optimal degree of exclusivity decreases with the number of competitors.

Finally, we consider two valuation distributions F and $\tilde{F}$ with density functions f and $\tilde{f}$. We assume that $\tilde{F}$ stochastically dominates F according to the likelihood ratio order: $\tilde{f}(x)/f(x) \leq \tilde{f}(y)/f(y)$ for $x \leq y$. We also assume that F, $1-F$, $\tilde{F}$ and $1-\tilde{F}$ are log-concave. We denote by $\bar{v}$ and $\tilde{\bar{v}}$ the optimal degrees of exclusivity and by b^* and $\tilde{b}^*$ the bidding functions under the distributions F and $\tilde{F}$. Finally we set $G = F^n$ and $\tilde{G} = \tilde{F}^n$.

We start with two observations. First, differentiating $\tilde{F}/F$ and using the likelihood ratio ordering property shows that the function $\tilde{F}/F$ is nondecreasing, hence $\tilde{F}(x)/\tilde{F}(y) \leq F(x)/F(y)$, and therefore $\tilde{G}(x)/\tilde{G}(y) \leq G(x)/G(y)$ for all $x \leq y$ and all $n \geq 1$. In other words, the distribution $\tilde{G}/\tilde{G}(y)$ first-order stochastically dominates the distribution $G/G(y)$ on $[0, y]$ for all y. From (1), it follows that the bidding functions are ranked $\tilde{b}^* \geq b^*$.

Second, multiplying $\tilde{f}(x)/f(x) \leq \tilde{f}(y)/f(y)$ and $\tilde{F}^{n-1}(x)/F^{n-1}(x) \leq \tilde{F}^{n-1}(y)/F^{n-1}(y)$ yields $\tilde{f}(x)\tilde{F}^{n-1}(x)/f(x)F^{n-1}(x) \leq \tilde{f}(y)\tilde{F}^{n-1}(y)/f(y)F^{n-1}(y)$. In other words, the distribution $\tilde{F}^n$ stochastically dominates the distribution F^n according to the likelihood ratio order. From standard arguments (two successive integrations), this property implies that for any $\bar{v}$ the left-truncated distribution $(\tilde{G}(v) - \tilde{G}(\bar{v}))/(1-\tilde{G}(\bar{v}))$ first-order stochastically dominates the distribution $(G(v) - G(\bar{v}))/(1-G(\bar{v}))$ on the interval $(\bar{v}, 1)$. Now we can write

$$0 = \int_{\bar{v}}^{1} [b^*(m) - \bar{v}] \frac{\mathrm{d}G(m)}{1-G(\bar{v})} \leq \int_{\bar{v}}^{1} \left[\tilde{b}^*(m) - \bar{v}\right] \frac{\mathrm{d}G(m)}{1-G(\bar{v})} \leq \int_{\bar{v}}^{1} \left[\tilde{b}^*(m) - \bar{v}\right] \frac{\mathrm{d}\tilde{G}(m)}{1-\tilde{G}(\bar{v})}.$$

The first equality is the definition of $\bar{v}$. The first inequality follows from $b^* \leq \tilde{b}^*$. The second inequality follows from the monotonicity of the function $\tilde{b}^*$ and the first-order stochastic ordering of the distributions $(\tilde{G}(v) - \tilde{G}(\bar{v}))/(1-\tilde{G}(\bar{v}))$ and $(G(v) - G(\bar{v}))/(1-G(\bar{v}))$ on the interval $(\bar{v}, 1)$. Because the problem is quasi-concave in $\bar{v}$, we conclude that $\tilde{\bar{v}} \geq \bar{v}$: there is less exclusivity under $\tilde{F}$ than under F.

Proof of Proposition 3 The two distributions F and F_0 satisfy the *Conditional Stochastic Dominance (CSD)* assumption presented in the above study,[14] which is a stronger property than first-order stochastic dominance.

Under partial exclusivity, $0 < \bar{v}_I < 1$, the joint profit of the IS-pair is v_I when $v_I \geq \bar{v}_I$. Otherwise an asymmetric auction takes place, where the strong player, E, has his valuation

[14] In the notations of Maskin and Riley (2000) page 419, $\beta_s = \beta_w = 0$, $\alpha_s = 1$, $\alpha_w = \bar{v}$, $\gamma = \bar{v}$, $\lambda = F(\bar{v})$.

distributed according to F on the whole interval $[0,1]$ while the weak player, I, has his valuation distributed according to the truncated distribution $F_I = F/F(\bar{v}_I)$ on the subinterval $[0, \bar{v}_I]$.

Let $[0, \bar{b}]$ be the support of both players' bid distributions and $\phi_I(b)$ denote the inverse bidding function of player I in this asymmetric auction. We therefore have: $\phi_I(\bar{b}) = \bar{v}_I$.

We also consider the (counterfactual) symmetric auction where bidder E is equally weak as bidder I, i.e. his valuation v_E is drawn from the truncated distribution F_I. We denote by $y_I(b)$ the inverse bidding function and by $[0, \mu_I]$ the support of the bid distribution in this symmetric auction (hence $y_I(\mu_I) = \bar{v}_I$). We know from Corollary 3.4 and Part (iii) of Proposition 3.5 in Maskin and Riley (2000) that $\mu_I \leq \bar{b}$ and $\phi_I(b) \leq y_I(b)$ for all $b \leq \mu_I$: as these authors put it, "if a weak bidder faces a strong bidder rather than a weak bidder he will bid more aggressively (closer to his valuation)". By monotonicity of the conditional expectation $\mathbb{E}(v|v \leq \bar{v})$, it follows that for all $b \leq \mu_I$

$$\frac{1}{F(\phi_I(b))} \int_0^{\phi_I(b)} v_I f(v_I) \mathrm{d}v_I \leq \frac{1}{F(y_I(b))} \int_0^{y_I(b)} v_I f(v_I) \mathrm{d}v_I = b, \tag{A.3}$$

where the right equality uses again the definition of the bid in a symmetric auction. For $\mu_I \leq b \leq \bar{b}$, the same inequality holds as

$$\frac{1}{F(\phi_I(b))} \int_0^{\phi_I(b)} v_I f(v_I) \mathrm{d}v_I \leq \frac{1}{F(\bar{v})} \int_0^{\bar{v}} v_I f(v_I) \mathrm{d}v_I = \mu_I \leq b. \tag{A.4}$$

Now, pick any value of v_E. When I wins the asymmetric auction, the IS-pair has the same payoff as under exclusivity. The interesting case is when E wins the auction, which happens if $v_I < \phi_I(b_E(v_E))$; in this case, the above inequality with $b = b_E(v_E)$ shows that the expected profit of the IS-pair, namely $F(\phi_I(b_E(v_E)))b_E(v_E)$, is higher than the expected value of v_I for $v_I \leq \phi_I(b_E(v_E))$, which yields the desired results.

Proof of Proposition 4 We start by studying the following asymmetric perturbation of the symmetric first-price auction with $n+1$ bidders: n strong bidders have their valuations uniformly distributed on $[0,1]$ and one weak bidder has his valuation uniformly distributed on $[0, \bar{v}]$, with $\bar{v}$ being equal or close to one. If $\bar{v} = 1$, the auction is symmetric, with all bidding functions being given by $b^*(v) = nv/(n+1)$ and all inverse bidding functions being given by $\phi^*(b) = (n+1)b/n$.

We are interested in the asymmetric auction where $\bar{v}$ is *slightly* below one. We denote by b_s and b_w the bidding functions of the strong and weak bidders respectively, and by ϕ_s and ϕ_w the corresponding inverse bidding functions. Because $\bar{v}$ is close to one, the changes in the bidding and inverse bidding functions relative to the symmetric auction are small. Compared to the symmetric auction, the n strong bidders reduce their bids while the weak bidder increases his bid, formally $\mathrm{d}b_s(v) = b_s(v) - b^*(v) < 0$ and $\mathrm{d}b_w(v) = b_w(v) - b^*(v) > 0$. Below we show that the first-order variations are inversely proportional to the number of the bidders: the reaction of the weak bidder is n times as large as that of the n strong bidders.

Lemma A.1 (Asymmetric perturbation of a symmetric first-price auction)**.** *Suppose that $\bar{v}$ is slightly below one. In absolute value, the changes in the bidding and inverse bidding functions relative to the symmetric auction satisfy:*

$$\mathrm{d}b_w = -n\,\mathrm{d}b_s > 0 \quad \textit{and} \quad \mathrm{d}\phi_w = -n\,\mathrm{d}\phi_s < 0. \tag{A.5}$$

Proof. The first-order condition of the weak bidder's problem are:

$$n\,\frac{\phi_s'(b)}{\phi_s(b)} = \frac{1}{\phi_w(b) - b} \tag{A.6}$$

while that of the strong bidder is

$$(n-1)\,\frac{\phi_s'(b)}{\phi_s(b)} + \frac{\phi_w'(b)}{\phi_w(b)} = \frac{1}{\phi_s(b) - b}. \tag{A.7}$$

For $i = s, w$, we consider the difference $y_i(b) = \mathrm{d}\phi_i(b) = \phi_i(b) - (n+1)b/n$ which is small when $\bar{v}$ is close to one. We observe that: $y_i(0) = 0$ for the two types of bidders. A first-order Taylor expansion of ϕ_i'/ϕ_i yields:

$$\frac{\phi_i'}{\phi_i} = \frac{1}{b}\left[1 + \frac{n}{n+1}\left(y_i' - \frac{y_i}{b}\right)\right].$$

Similarly a first-order Taylor expansion of $1/(\phi_i - b)$ yields

$$\frac{1}{\phi_i - b} = \frac{n}{b}\left(1 - \frac{n y_i}{b}\right).$$

After simplification by n/b, we get from (A.6) that

$$y_s' - \frac{y_s}{b} = -(n+1)\frac{y_w}{b}.$$

and from (A.7) that

$$\frac{n-1}{n+1}\left(y_s' - \frac{y_s}{b}\right) + \frac{1}{n+1}\left(y_w' - \frac{y_w}{b}\right) = -n\frac{y_s}{b}.$$

Combining the latter two equations yields $z' = -nz/b$ where the function z is defined by $z(b) = y_s(b) + y_w(b)/n$. As $z(0) = 0$, we find that z is identically zero, hence $y_w = -ny_s$, the desired result for the perturbation of the inverse bid functions. By definition of the bid and inverse bid functions, we have that: $\phi_i(b_i(v; \bar{v}); \bar{v}) = v$. Differentiating with $\bar{v}$ at $\bar{v} = 1$, we get $\frac{n+1}{n}\,\mathrm{d}b_i + \mathrm{d}\phi_i = 0$, hence

$$\mathrm{d}b_i = -n\,\mathrm{d}\phi_i\,/(n+1), \tag{A.8}$$

which yields the result for the perturbations of the bid functions. □

We are now able to prove Proposition 4. We suppose that F is uniform on $[0, 1]$ and compare full competition ($\bar{v} = 1$) with partial exclusivity, $\bar{v}$ slightly lower to one.

Let $m = \max(v_1, \cdots, v_n)$ be the maximal valuation of the competitors. Compared to full competition ($\bar{v} = 1$), partial exclusivity has two effects. On the one hand, for any given value of m, partial exclusivity induces a loss for the seller-preferred buyer coalition in the form of a reduction in the bids placed by competitors. As the competitors win with probability close to $\Pr(v_0 \leq m) = m$, this negative effect on the coalition's profit is

$$\delta_1 = m\,\mathrm{d}b_s(m) < 0.$$

On the other hand, as $\bar{v}$ decreases, the probability that the coalition deals internally increases for two reasons: first because that buyer increases his bid and second because the competitors decrease their bids. For any value of m, the probability that the coalition deals internally is

$$\Pr(B_0 \text{ gets the good } |m) = 1 - \Pr(b_s(m; \bar{v}) \geq b_w(v_0; \bar{v})) = 1 - \phi_w(b_s(m; \bar{v}); \bar{v}),$$

which, as mentioned above, is close to m when $\bar{v}$ is close to one. This probability depends on $\bar{v}$ both directly and indirectly through $b_s(m; \bar{v})$. Differentiating with respect to $\bar{v}$ at $\bar{v} = 1$ yields

$$\mathrm{d}.\Pr(B_0 \text{ gets the good } |m) = -\mathrm{d}\phi_w - \frac{n+1}{n}\mathrm{d}b_s > 0,$$

with both $\mathrm{d}\phi_w$ and $\mathrm{d}b_s$ being negative.[15] This second effect benefits the coalition because in this region it now earns m instead of $nm/(n + 1)$. Hence a positive effect on the coalition's profit

$$\delta_2 = -\left\{\mathrm{d}\phi_w + \frac{n+1}{n}\mathrm{d}b_s\right\}\frac{m}{n+1} > 0. \tag{A.9}$$

Applying Lemma A.1, we get that the first component of δ_2 is

$$-\mathrm{d}\phi_w\frac{m}{n+1} = n\,\mathrm{d}\phi_s\frac{m}{n+1} = -m\,\mathrm{d}b_s = -\delta_1,$$

where the second equality above uses (A.8). It follows that the net effect $\delta_1 + \delta_2$ is positive, the desired result.

Proof of Proposition 5 We first consider the asymmetric auction with one weak bidder and n strong bidders, $n \geq 2$. The strong bidders have i.i.d. valuations drawn from the distribution F with support on $[0, 1]$. The weak bidder has his valuation drawn in the right-truncated $F/F(\bar{v})$ on $[0, \bar{v}]$. We denote by $b_s = b_1 = \cdots = b_n$ the common bidding function of the strong bidders. The support of the distribution of bids placed by the weak bidder and the strong bidders are respectively the interval $[0, b_0(\bar{v})]$ and the interval $[0, b_s(1)]$.

[15] To obtain the second term, we have used $\phi_w(b; 1) = (n + 1)b/n$.

When there is one competitor ($n = 1$), the two intervals are the same, $b_0(\bar{v}) = b_s(1)$, regardless of the value of $\bar{v}$. As explained at the beginning of section 5, the above equality implies that $b_s(1) < \bar{v}$, which gives the coalition an incentive to deviate towards a higher degree of exclusivity (a lower threshold $\bar{v}$) than expected by the competitors. Doing so, the coalition deals internally in a slightly expanded region, where it earns $\bar{v}$ instead of $b_s(1)$.

The identity of the supports of the weak and strong b

Lemma 2. *Suppose there are at least two competing buyers, $n \geq 2$. Then there exists a threshold $\hat{v}_n$ in $(0, 1)$ such that*

- *if $\bar{v} \geq \hat{v}_n$, the weak bidder's and strong bidders' maximal bids coincide: $b_s(1) = b_0(\bar{v}) \geq b_n^*(1)$;*
- *if $\bar{v} < \hat{v}_n$, the weak bidder's maximal bid is lower than the strong bidders' maximal bid: $b_0(\bar{v}) < b_s(1) = b_n^*(1)$. Strong bidders with valuations above $\phi_n^*(b_0(\bar{v}))$ use the bidding function b_n^* as if the weak bidder were absent.*

The threshold $\hat{v}_n$ is such that: $b_0(\hat{v}_n) = b_n^*(1)$. We have $b_{n+1}^*(\hat{v}_n) < b_0(\hat{v}_n) < \hat{v}_n$ because the weak bidder bids more aggressively than in the symmetric auction with $n+1$ bidders. It follows that $0 < b_n^*(1) < \hat{v}_n < \phi_{n+1}^*(b_n^*(1)) < 1$.

When $\bar{v}^e$ is higher than $\hat{v}_n$, the distribution of bids have the same support for all players, and as a result the highest bid placed by the competitors is strictly lower than the exclusivity threshold: the unraveling reasoning exposed in the one competitor case (see the beginning of section 5) applies: deviating to a slightly more exclusive agreement is profitable for the coalition.

Suppose now $\bar{v}^e \geq \hat{v}_n$. The probability that the indifferent preferred buyer $v_0 = \bar{v}^e$ at the candidate equilibrium is $G_n(\phi_n^*(b_0(\bar{v}^e))) < 1$. The indifference condition yields the strike price $\bar{b}^e$:

$$\bar{v}^e - \bar{b}^e = G_n(\phi_n^*(b_0(\bar{v}^e))) \left[\bar{v}^e - b_0(\bar{v}^e)\right].$$